Student Number

St. Catherine's School Waverley

August 2014

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16
- Task weighting – 45%

Total Marks – 100

Section I **Pages 3-5**

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided.

Section II **Pages 6-13**

90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section
- Answer each question in the booklet provided.

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Section I

Total marks - 10
Attempt Questions 1-10
All questions are of equal value.

Answer either *A,B,C* or *D* on the multiple choice answer sheet provided.

Q1. Factorise $x^2y - xy^2 - x + y$.

(A) $(xy-1)(x+y)$ (B) $(xy-1)(x-y)$

(C) $(xy+1)(x+y)$ (D) $(xy+1)(x-y)$

Q2. Find $\int (2x+1)^5 dx$

(A) $\dfrac{(2x+1)^6}{3} + C$ (B) $\dfrac{(2x+1)^6}{6} + C$

(C) $\dfrac{(2x+1)^6}{12} + C$ (D) $\dfrac{5(2x+1)^4}{4} + C$

Q3. Find the limiting sum of the geometric series

$$1 + \frac{\sqrt{2}}{\sqrt{2}+1} + \frac{2}{(\sqrt{2}+1)^2} + \frac{2\sqrt{2}}{(\sqrt{2}+1)^3} + \ldots..$$

(A) $\sqrt{2}+1$ (B) $\sqrt{2}-1$

(C) $-\sqrt{2}-1$ (D) $\dfrac{1}{\sqrt{2}+1}$

Q4. For what values of k does the quadratic equation $kx^2 + kx + 1 = 0$ have no real roots?

(A) $k < 0 \text{ or } k < 4$ (B) $0 < k < 4$

(C) $k > 4$ (D) $k < 0 \text{ or } k > 4$

Q5. Solve the equation $\sqrt{3}\tan x + 3 = 0$ for $0 \le x \le 2\pi$

(A) $x = \dfrac{5\pi}{6}, \dfrac{11\pi}{3}$ (B) $x = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$

(C) $x = \dfrac{2\pi}{3}, \dfrac{5\pi}{3}$ (D) $x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$

Q6. The table below shows the values of a function *f(x)* for five values of *x*.

x	2	2.5	3	3.5	4
f(x)	4	1	-2	3	8

What value is an estimate for $\int_2^4 f(x)dx$ using Simpson's Rule with five function values?

(A) 12 (B) 6

(C) 8 (D) 4

Q7. If α and β are the roots of $2x^2 + 3x - 6 = 0$, what is the value of $\dfrac{\alpha\beta}{\alpha+\beta}$?

(A) 2 (B) $\dfrac{1}{2}$

(C) $-\dfrac{1}{2}$ (D) -2

Q8.

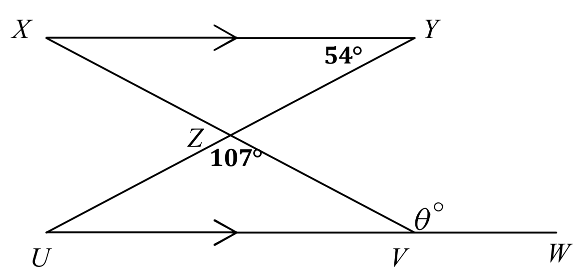


FIGURE NOT TO SCALE

The diagram above shows XY parallel to UW, $\angle XYU = 54°$, $\angle UZV = 107°$ and $\angle ZVW = \theta°$. The value of θ is:

(A) 161 (B) 19

(C) 54 (D) 107

Q9. What are the solutions to the equation $2^{6x} - 9(2^{3x}) + 8 = 0$?

(A) $x = 1 \ or \ x = 8$

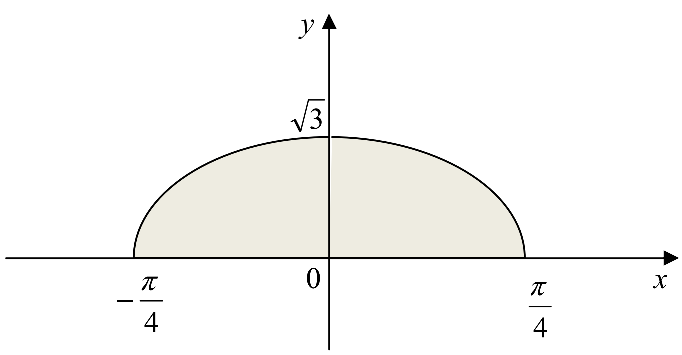
(B) $x = 0 \ or \ x = \frac{8}{3}$

(C) $x = 0 \ or \ x = 1$

(D) $x = 1 \ or \ x = \frac{1}{6}$

Q10. The diagram shows the region bounded by the curve $y = \sqrt{3\cos 2x}$ and the x-axis for $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.

The region is rotated about the $x-$axis to form a solid.



Which of the following gives the volume of the solid?

(A) $V = 3\pi \int_0^{\frac{\pi}{4}} \cos 2x \ dx$

(B) $V = 9\pi \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \cos 2x \ dx$

(C) $V = 6\pi \int_0^{\frac{\pi}{4}} \cos 4x \ dx$

(D) $V = 6\pi \int_0^{\frac{\pi}{4}} \cos 2x \ dx$

End of Section I

Section II

Total marks - 90
Attempt Questions 11-16
All questions are of equal value
Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use the Question 11 Writing Booklet **Marks**

(a) Simplify $\frac{2x}{3}-\frac{3x+8}{12}$ /1

(b) Solve $|x-3|\le 7$ /2

(c) If $h(x)=\begin{cases} ax+1 & \text{if } x\le 1 \\ x^2-5 & \text{if } x>1 \end{cases}$ and $h(-2)=h(4)$, find a. /2

(d) If $(3+\sqrt{5})^2=a+\sqrt{b}$, find a and b. /2

(e) In the diagram below $BC=1$ unit, $\angle BCA=60^\circ$ and $\angle CDA=30^\circ$.

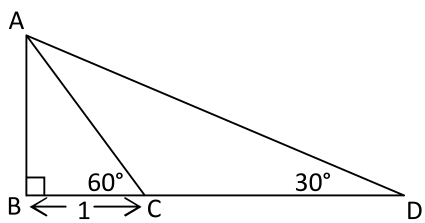


FIGURE NOT TO SCALE

(i) Find the exact length of AB /1

(ii) Hence, or otherwise, find the length of *CD*. /2

(f) The first term of a geometric series is 4 and the eighth term is 8748.
Find the twelfth term. /2

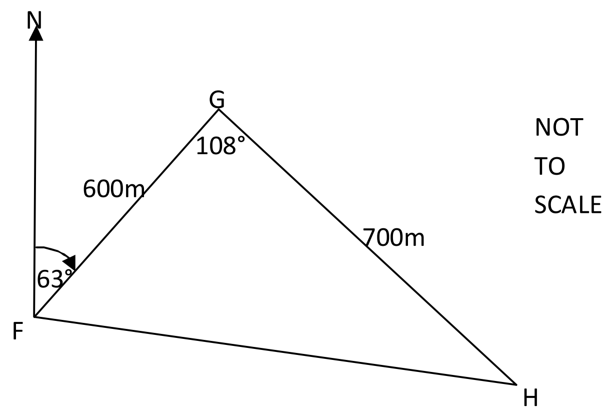
(g) Find the equation of the normal to the curve $y=\tan x$ at the point $(\frac{\pi}{3},\sqrt{3})$. /3

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) Jenny is setting up part of an orienteering course.

She follows the course from F to G to H as shown in the diagram below.

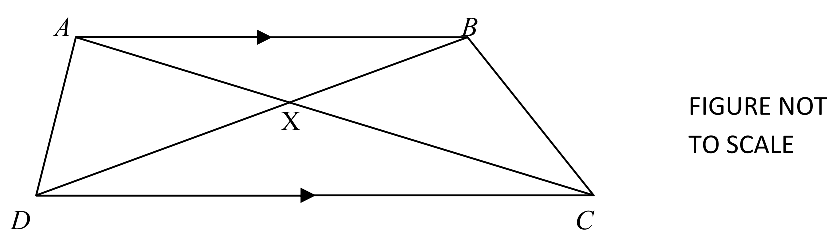


(i) Show that the distance FH to the nearest metre is 1053 metres. /2

(ii) Hence, or otherwise, calculate the size of $\angle GFH$ to the nearest degree. /2

(iii) If the bearing of G from F is 063° calculate the bearing of H from F to the nearest degree. /1

(b) *ABCD* is a trapezium, and *AB* is parallel to *DC*.
The diagonals *AC* and *BD* intersect at *X*. $AX = 4$cm, $CX = 10$cm, $BD = 12$cm.



Copy the diagram neatly into your answer booklet.

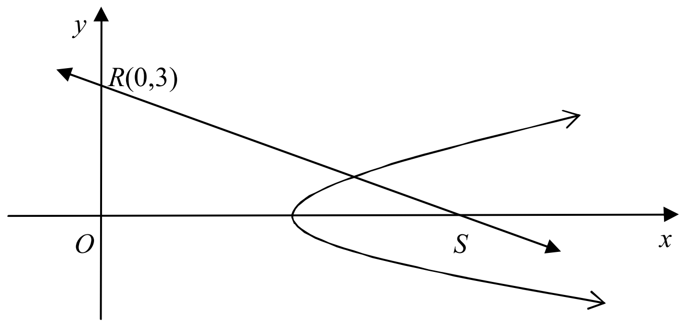
(i) Prove ΔAXB is similar to ΔCXD. /2

(ii) Find the length of *BX*. /2

Question 12 continues on page 8

Question 12 continued

(c) RS is a straight line where R has co-ordinates $(0,3)$ and S is the focus of the parabola .
$y^2 = 4x - 12$.



(i) Show that the co-ordinates of the focus S are $(4,0)$. /2

(ii) Show that the equation of the line RS is $3x + 4y - 12 = 0$ /1

(iii) Find the perpendicular distance from a point $P(0,\frac{1}{2})$ to the line RS. /2

(iv) Hence, write the equation of the circle that has its centre at P and has RS as a tangent /1

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) For the equation $y = \frac{1}{4}x^2 + 2x - 1$

(i) Show that $(x+4)^2 = 4(y+5)$ /2

(ii) Hence, or otherwise, find:

(α) the coordinates of the vertex /1

(β) the equation of the directrix /1

(b) Show that $\cos ec\theta - \sin\theta = \cot\theta \cos\theta$ /2

(c) Find the volume obtained by rotating about the x axis the area beneath the curve $y = e^x$ from $x = 0$ to $x = 2$. Give your answer as an exact value. /3

(d) If $1, a, b$ form an arithmetic sequence and $1, b, a$ form a geometric sequence and $a \neq b$:

(i) Show that $2a - b = 1$ and $a = b^2$ /1

(ii) Hence, or otherwise, find a and b. /2

(e) (i) On the same set of axes, sketch the graphs of the functions

$y = 2\sin x$ and $y + 1 = 0$ where $-\pi \leq x \leq \pi$. /2

(ii) Hence, or otherwise, find the number of solutions for $\sin x = -\frac{1}{2}$ in the same domain. /1

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Find:

(i) $\dfrac{dy}{dx}$ if $y = x^2 \log_e x$ /2

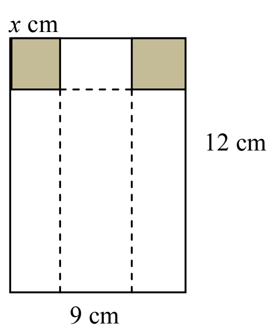
(ii) $\displaystyle\int \frac{\sin x}{1+\cos x}dx$ /2

(b) (i) Differentiate $e^{tan\,x}$ /1

(ii) Hence, or otherwise, evaluate $\displaystyle\int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x \; dx$. /2

(c) Find the value of *m* for which $x^2 + (m-1)x - m = 0$ has equal roots /3

(d) A rectangular sheet of cardboard measures 12cm by 9cm. From two corners, squares of side *x* cm are removed as shown. The remainder is folded along the dotted lines to form a tray as shown.



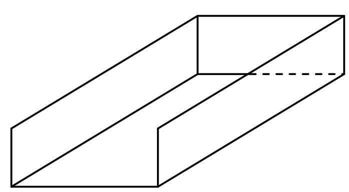


FIGURE NOT TO SCALE

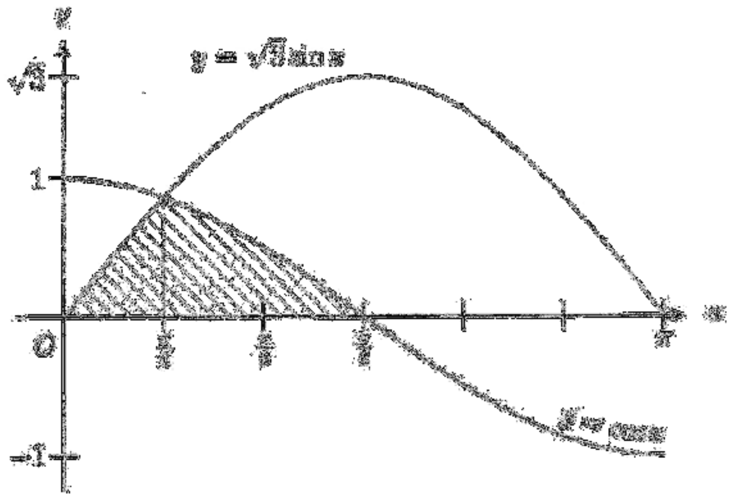
(i) Show that the volume, V cm^3, of the tray is given by $V = 2x^3 - 33x^2 + 108x$. /2

(ii) Find the maximum volume of the tray /3

End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) The diagram below shows the graphs of the functions $y = \sqrt{3}\sin x$ and $y = \cos x$ between $x = 0$ and $x = \pi$. The two graphs intersect at $x = \dfrac{\pi}{6}$.



Show that the area of the shaded region between $x = 0$ and $x = \dfrac{\pi}{2}$ is $(\sqrt{3} - 1)$ square units. /3

(b) At the beginning of 2008 the population N of birds on an island was 10 000.

At the beginning of 2012 this population was 160 000.

Assume that the population N was increasing exponentially and satisfies the equation $N = Ae^{kt}$, where A and k are constants and the time t is measured in years from the beginning of 2007.

(i) Find the constants A and k. /3

(ii) Find the time t, to the nearest year, required for the population N to reach 1.28 million. /2

Question 15 continues on page 12

Question 15 continued

(c) The velocity v in ms^{-1} of a particle moving in a straight line is given by:

$v = 36 - 4e^{2t}$ where t is the time in seconds and d is the distance travelled in metres.

(i) Find the initial velocity of the particle. /1

(ii) Show that the exact time at which the particle first comes to rest is $t = \log_e 3$. /2

(iii) Find the distance travelled by the particle during this time. /2

(iv) Find an expression for the acceleration a in terms of v. /2

End of Question 15

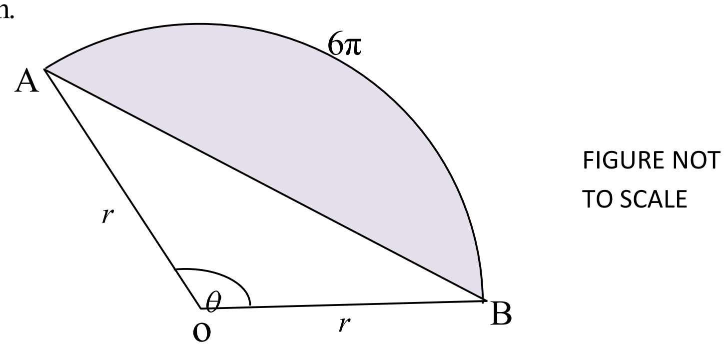
Question 16 (15 marks) Use the Question 16 Writing Booklet.

(a) Water is flowing into a container which can be filled to a depth (D) of 900 millimetres. When the water began to flow, the container already held water to a depth of 150 millimetres. The rate, at which the depth of water in the container is increasing, in millimetres per minute, is given by $R = 6t + 5, (t \geq 0)$. Find:

(i) the depth of water in the container after eight minutes /3

(ii) the time it takes to fill the container. /2

(b) The diagram shows a sector with angle θ at the centre and radius r cm, where $6 < r < 12$. The arc length is 6π cm.



(i) Show why θ is an obtuse angle /2

(ii) Calculate the area of the shaded minor segment when $\theta = \frac{3\pi}{4}$ radians.

(Hint: first find the value of r) /2

(c) Given the function $y = \frac{10}{3 + 2\sin x}$ in the domain $0 \leq x \leq 2\pi$:

(i) Show that $\frac{dy}{dx} = \frac{-20\cos x}{(3 + 2\sin x)^2}$ /1

(ii) Show that there are stationary points at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ and describe their nature. /3

(iii) Graph the function in the given domain showing all essential features /2

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \; if \; n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Student Number

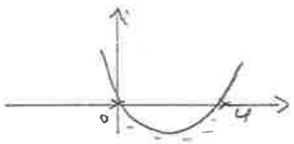
YEAR 12 TRIAL HSC MATHEMATICS 2014
MULTIPLE CHOICE ANSWER SHEET

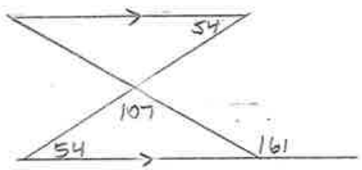
Section I - Mark your answer in the appropriate box with an X.

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Begin Section II using your writing booklets

2 UNIT MATHEMATICS TRIAL 2014

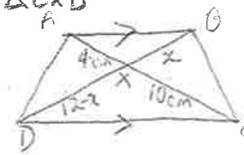
Qn	MC Solutions	Marks	Comments: Criteria
1.	$x^2y - xy^2 - x + y$ $= xy(x-y) - (x-y)$ $= (x-y)(xy-1)$ B		
2.	$\int (2x+1)^5 \, dx$ $= \frac{(2x+1)^6}{6\times 2} + c$ $= \frac{(2x+1)^6}{12} + c$ C		
3.	$1 + \frac{\sqrt{2}}{\sqrt{2}+1} + \frac{2}{(\sqrt{2}+1)^2} + \frac{2\sqrt{2}}{(\sqrt{2}+1)^3} + \cdots$ $a = 1 \quad r = \frac{\sqrt{2}}{\sqrt{2}+1}$ $S_\infty = \frac{a}{1-r}$ $= \frac{1}{1-\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right)}$ $= \frac{1}{\frac{(\sqrt{2}+1)-\sqrt{2}}{\sqrt{2}+1}}$ $= \sqrt{2}+1$ A		
4.	$kx^2 + kx + 1 = 0$ $a = k \quad b = k \quad c = 1$ $b^2 - 4ac < 0$ $k^2 - 4 \times k \times 1 < 0$ $k^2 - 4k < 0$ $k(k-4) < 0$ $\therefore \; 0 < k < 4$  B		

Qn	Solutions	Marks	Comments: Criteria
5.	$\sqrt{3}\tan x + 3 = 0$ $\sqrt{3}\tan x = -3$ $\tan x = \frac{-3}{\sqrt{3}}$ $= -\frac{3\sqrt{3}}{3}$ $= -\sqrt{3}$ acute angle $= \frac{\pi}{3}$ $\therefore \theta = \pi - \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}$ $= \frac{2\pi}{3}, \; \frac{5\pi}{3}$ C		
6.	$A \doteqdot \frac{0.5}{3}\{4 + 4(1+3) + 2(-2) + 8\}$ $= \frac{1}{6}\{4 + 16 - 4 + 8\}$ $= \frac{1}{6}\{24\}$ $= 4$ D		
7.	$2x^2 + 3x - 6 = 0$ $a = 2 \quad b = 3 \quad c = -6$ $\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$ $\alpha\beta = \frac{c}{a} = \frac{-6}{2} = -3$ $\therefore \frac{\alpha\beta}{\alpha+\beta} = \frac{-3}{-\frac{3}{2}}$ $= -3 \times \frac{2}{-3}$ $= 2$ A		
8.	 $\theta = 161^\circ$ A		

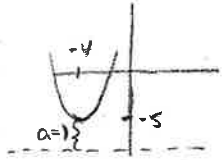
Qn	Solutions	Marks	Comments: Criteria
9.	$2^{6x} - 9(2^{3x}) + 8 = 0$ ie $(2^{3x})^2 - 9(2^{3x}) + 8 = 0$ let $u = 2^{3x}$ $u^2 - 9u + 8 = 0$ $(u-8)(u-1) = 0$ $2^{3x} - 8 = 0$ or $2^{3x} - 1 = 0$ $2^{3x} = 8$ $2^{3x} = 1$ $2^{3x} = 2^3$ $2^{3x} = 2^0$ $3x = 3$ $3x = 0$ $\underline{x=1}$ $\underline{x=0}$ C		
10.	$V = \pi \int_a^b y^2\, dx$ $= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sqrt{3\cos 2x})^2\, dx$ $= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 3\cos 2x\, dx$ $= 3\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x\, dx$ $= 2 \times 3\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x\, dx$ $= 6\pi \int_0^{\frac{\pi}{4}} \cos 2x\, dx$ D		

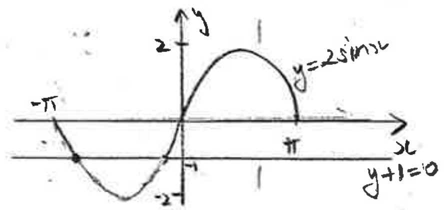
Qn 11	Solutions	Marks	Comments: Criteria
(a)	$\frac{2x}{3} - \frac{3x+8}{12} = \frac{8x-3x-8}{12}$ $= \frac{5x-8}{12}$	1	$\frac{1}{2}$ for $\frac{8x-3x+8}{12} = \frac{5x+8}{12}$
(b)	$\|x-3\| \le 7$ $-7 \le x-3 \le 7$ 1 $-4 \le x \le 10$ 1 $x-3 \le 7$ or $-(x-3) \le 7$ $x \le 10$, $-x+3 \ge -7$ $x \ge -4$ 1 $-4 \le x \le 10$	2	
(c)	$h(-2) = -2a+1$ $\frac{1}{2}$ $h(4) = 4^2-5 = 11$ $\frac{1}{2}$ if $h(-2) = h(4)$ $-2a+1 = 11$ $\frac{1}{2}$ $-2a = 10$ $a = -5$ $\frac{1}{2}$	2	
(d)	$(3+\sqrt{5})^2 = 9 + 6\sqrt{5} + 5$ $= 14 + 6\sqrt{5}$ 1 $= 14 + \sqrt{180}$ $\therefore a = 14$ $\frac{1}{2}$ $b = 180$ $\frac{1}{2}$	2	
(e) i)	$\tan 60° = \frac{AB}{1}$ 1/2 $\therefore AB = \sqrt{3}$ 1/2	1	
ii)	$\tan 30 = \frac{AB}{BD}$ $\frac{1}{2}$ $BD = \frac{\sqrt{3}}{\tan 30}$ $\frac{1}{2}$ $= \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$ $= 3$ $\frac{1}{2}$ $\therefore CD = BD - BC$ $= 3-1$ $= 2$ units $\frac{1}{2}$	2	
(f)	$a = 4$ $ar^7 = 8748$ 1/2 $4r^7 = 8748$ $\frac{1}{2}$ $r^7 = 2187$ $r = 3$ $\frac{1}{2}$ $\therefore T_{12} = ar^{11}$ $= 4 \times 3^{11}$ $= 708588$ $\frac{1}{2}$	2	$\frac{1}{2}$/2 IF DONE AS AN A.P.

Qn	Solutions	Marks	Comments: Criteria
	11 cont		
(g)	$y = \tan x$ $\frac{dy}{dx} = \sec^2 x$ 1 at $(\frac{\pi}{3}, \sqrt{3})$ $\frac{dy}{dx} = \sec^2(\frac{\pi}{3}) = (2)^2 = 4$ $\frac{1}{2}$ $\therefore m_\perp = -\frac{1}{4}$ $\frac{1}{2}$ $y - y_1 = m(x - x_1)$ $y - \sqrt{3} = -\frac{1}{4}(x - \frac{\pi}{3})$ $\frac{1}{2}$ $4y - 4\sqrt{3} = -x + \frac{\pi}{3}$ $x + 4y - 4\sqrt{3} - \frac{\pi}{3} = 0$ $3x + 12y - 12\sqrt{3} - \pi = 0$ $\frac{1}{2}$ (OR $y = -\frac{x}{4} + \sqrt{3} + \frac{\pi}{12}$)	3	

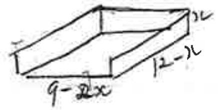
Qn	Solutions	Marks	Comments: Criteria
12			
(i)	$FH^2 = 600^2 + 700^2 - 2 \times 600 \times 700 \cos 108$ 1 $= 1109574.275$ $FH = 1053$ (nearest metre) 1	2	
(ii)	$\frac{\sin\theta}{700} = \frac{\sin 108}{1053}$ 1 $\sin\theta = \frac{\sin 108}{1053} \times 700$ $\therefore \theta = 39°$ 1	2	
(iii)	bearing of H from F is $63 + 39 = 102°$ 1	1	
(b) (i)	In ΔAXB and ΔCXD $\angle BAX = \angle DCX$ (alternate $\angle$, $AB \parallel DC$) $\frac{1}{2}$ $\angle ABX = \angle CDX$ (alternate $\angle$, $AB \parallel DC$) $\frac{1}{2}$ $\angle AXB = \angle CXD$ (vert. opp) $\therefore \Delta AXB \parallel\!\mid \Delta CXD$ (equiangular) $\frac{1}{2}$	2	
(ii)	Since $\Delta AXB \parallel\!\mid \Delta CXD$  $\frac{x}{4} = \frac{12-x}{10}$ 1 $10x = 48 - 4x$ $\frac{1}{2}$ $14x = 48$ $x = \frac{24}{7}$ $\frac{1}{2}$ OR $\frac{BX}{12} = \frac{4}{14}$ $14BX = 48$ $BX = \frac{24}{7}$	2	1 FOR $\frac{BX}{DX} = \frac{4}{10}$
(c) (i)	$y^2 = 4x - 12$ $y^2 = 4(x - 3)$ Vertex is $(3, 0)$ 1 $4a = 4$ $a = 1$ $\therefore$ focus is $(4, 0)$ 1	2	
(ii)	$m_{RS} = -\frac{3}{4}$ 1/2 $y - 0 = -\frac{3}{4}(x - 4)$ 1/2 $4y = -3x + 12$ $3x + 4y - 12 = 0$	1	

Qn	Solutions	Marks	Comments: Criteria
12			
c) iii)	$d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2+b^2}}$ $a=3 \quad b=4 \quad c=-12$ $(x_1,y_1) = (0,\frac{1}{2})$ $= \frac{\lvert 3(0)+4(\frac{1}{2})-12 \rvert}{\sqrt{3^2+4^2}}$ 1 $= \frac{\lvert -10 \rvert}{\sqrt{25}}$ $= 2$ 1	2	
iv)	circle : $(x-0)^2 + (y-\frac{1}{2})^2 = 4$ 1	1	$\frac{1}{2}$ FOR WRONG RADIUS

Qn	Solutions	Marks	Comments: Criteria
13			
(a)	$y = \frac{1}{4}x^2 + 2x - 1$		
(i)	$4y = x^2 + 8x - 4$ $4y + 4 = x^2 + 8x$ 1/2 $4y + 4 + (\frac{8}{2})^2 = x^2 + 8x + (\frac{8}{2})^2$ $4y + 20 = (x+4)^2$ 1 $\therefore (x+4)^2 = 4(y+5)$ $\frac{1}{2}$	2	
(ii)	α) vertex $= (-4,-5)$ 1	1	
	β) $4a = 4$ $a = 1$ (focal length) $\therefore$ directrix : $y = -6$ 1 	1	
(b)	$\operatorname{cosec}\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta$ $= \frac{1-\sin^2\theta}{\sin\theta}$ 1 $= \frac{\cos^2\theta}{\sin\theta}$ $\frac{1}{2}$ $= \frac{\cos\theta}{\sin\theta} \times \cos\theta$ $\frac{1}{2}$ $= \cot\theta \cos\theta$	2	
(c)	$A = \pi \int_0^2 y^2 \, dx$ $= \pi \int_0^2 (e^x)^2 \, dx$ 1/2 $= \pi \int_0^2 e^{2x} \, dx$ $\frac{1}{2}$ $= \pi \left[\frac{e^{2x}}{2} \right]_0^2$ 1 $= \pi \left(\frac{e^4}{2} - \frac{e^0}{2} \right)$ $= \pi \left(\frac{e^4}{2} - \frac{1}{2} \right) u^3$ 1	3	

Qn	13 Solutions	Marks	Comments: Criteria
(d)(i)	$a-1=b-a$ ½ ① $\therefore 2a-b=1$ $\frac{b}{1}=\frac{a}{b}$ ½ $b^2=a$ ②	1	
(ii)	sub ② into ① $2(b^2)-1=b$ ½ $2b^2-b-1=0$ $(2b+1)(b-1)=0$ ½ $b=-\frac{1}{2}, 1$ ½ since $a\neq b$, $b\neq 1$ $\therefore b=-\frac{1}{2}$ and $a=\frac{1}{4}$ ½	2	$-\frac{1}{2}$ if both solutions are given.
(e)(i)		2	
(ii)	$2\sin x=-1$ $\sin x=-\frac{1}{2}$ $\therefore$ 2 solutions 1	1	

Qn	14 Solutions	Marks	Comments: Criteria
(a)(i)	$y=x^2\log_e x$ $u=x^2$, $\frac{du}{dx}=2x$ $v=\log_e x$, $\frac{dv}{dx}=\frac{1}{x}$ ✓ $\frac{dy}{dx}=2x\log_e x+x^2\times\frac{1}{x}$ $=2x\log_e x+x$ $=x(2\log_e x+1)$ ✓	2	
(ii)	$\int\frac{\sin x}{1+\cos x}\,dx=-\int\frac{-\sin x}{1+\cos x}\,dx$ $=-\log_e(1+\cos x)+c$ ✓ ✓	2	1 for log 1 for minus
(b)(i)	$\frac{d}{dx}(e^{\tan x})=\sec^2 x\tan x$ ✓	1	
(ii)	$\int_0^{\frac{\pi}{4}}\frac{d}{dx}e^{\tan x}\,dx=\int_0^{\frac{\pi}{4}}\sec^2 x\tan x\,dx$ $e^{\tan x}\Big]_0^{\frac{\pi}{4}}=\int_0^{\frac{\pi}{4}}\sec^2 x\tan x\,dx$ ✓ $e^{\tan\frac{\pi}{4}}-e^{\tan 0}=$ ½ $e^1-e^0=$ $\therefore\int_0^{\frac{\pi}{4}}\sec^2 x\tan x\,dx=e-1$ ½	2	1 for $e^{\tan x}\Big]_0^{\frac{\pi}{4}}$ ½ for sub. ½ for answer
(c)	$x^2+(m-1)x-m=0$ $\alpha+\alpha=-\frac{b}{a}$ $\alpha\times\alpha=\frac{c}{a}$ $2\alpha=\frac{-(m-1)}{1}$ $\alpha^2=\frac{-m}{1}$ ② ½ $2\alpha=1-m$ $\alpha=\frac{1-m}{2}$ ① ½ sub ① into ② $\left(\frac{1-m}{2}\right)^2=-m$ ✓ $\frac{1-2m+m^2}{4}=-m$ $1-2m+m^2=-4m$ $m^2+2m+1=0$ $(m+1)^2=0$ $\therefore m=-1$ ✓ OR $b^2-4ac=0$ $(m-1)^2-4\times1\times-m=0$ ✓ $m^2-2m+1+4m=0$ $m^2+2m+1=0$ ✓ $(m+1)^2=0$ $m=-1$ ✓	3	–

Qn	14 Solutions	Marks	Comments: Criteria
d) (i)	$V = x(12-x)(9-2x)$ ✓ $= x(108 - 33x + 2x^2)$ $V = 2x^3 - 33x^2 + 108x$ ✓  	2	
(ii)	$\frac{dV}{dx} = 6x^2 - 66x + 108$ ✓ $\frac{dV}{dx} = 0$ (for max) $6(x^2 - 11x + 18) = 0$ $x^2 - 11x + 18 = 0$ $(x-9)(x-2) = 0$ $\therefore x = 9, 2$ but $x \neq 9$ $\therefore x = 2$ ✓ $\frac{d^2V}{dx^2} = 12x - 66$ at $x=2$ $\frac{d^2V}{dx^2} = 12(2) - 66$ < 0 $\therefore x = 2$ is a maximum $\therefore$ max volume $= 2(12-2)(9-2(2))$ ½ $= 100 \text{ cm}^3$	3	1 for diff 1 for x values. ½ for checking for max ½ for sub of x=2 to get answer

7 part marks

Qn	15 Solutions	Marks	Comments: Criteria
(a)	$A = \int_0^{\frac{\pi}{6}} \sqrt{3}\sin x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \, dx$ ✓ $= -\sqrt{3}\cos x\Big]_0^{\frac{\pi}{6}} + \sin x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ ✓ $= -\sqrt{3}(\cos\frac{\pi}{6} - \cos 0) + (\sin\frac{\pi}{2} - \sin\frac{\pi}{6})$ ½ $= -\sqrt{3}\left(\frac{\sqrt{3}}{2} - 1\right) + \left(1 - \frac{1}{2}\right)$ $= -\frac{3}{2} + \sqrt{3} + \frac{1}{2}$ $= \sqrt{3} - 1$ sq units ½	3	Only 1 mark if $\int_0^{\frac{\pi}{2}} \sqrt{3}\sin x - \cos x$ two marks if $\int_0^{\frac{\pi}{2}} \sqrt{3}\sin x \, dx - \int_0^{\frac{\pi}{2}} \cos x$
(b) (i)	$N = Ae^{kt}$ when $t=1$ $N = 10000$ $10000 = Ae^{k(1)}$ $\therefore A = \frac{10000}{e^k}$ ① ½ when $t=5$ $N = 160000$ $160000 = Ae^{5k}$ $160000 = \frac{10000}{e^k} \times e^{5k}$ ✓ $\frac{160000}{10000} = e^{4k}$ $16 = e^{4k}$ $\log_e 16 = 4k \log_e e$ $k = \frac{\log_e 16}{4}$ ½ $= 0.6931$ ② $\therefore A = \frac{10000}{e^{0.6931}}$ $A = 5000$ ✓	3	1 for correct A 2 for correct k.
(ii)	$128000 = 5000e^{0.6931t}$ ✓ $\frac{128000}{5000} = e^{0.6931t}$ $\log_e 256 = \log_e e^{0.6931t}$ ½ $\frac{\log_e 256}{0.6931} = t$ $t = 8$ years ½	2	

Qn	15 Cont Solutions	Marks	Comments: Criteria
(c) (i)	$V = 36 - 4e^{2t}$ ~~$t=0, v = 36-4e^{2(0)}$~~ ✓ $V = 36 \times 4(1)$ $= 32 m/s$	1	
(ii)	when $v=0$ $36 - 4e^{2t} = 0$ ✓ $\frac{1}{2}$ $36 = 4e^{2t}$ $\frac{1}{2}$ $9 = e^{2t}$ $\log_e 9 = \log_e e^{2t}$ $\frac{1}{2}$ $\log_e 9 = 2t \log_e e$ $\frac{\log_e 9}{2} = t$ $\frac{2\log_e 3}{2} = t$ $\frac{1}{2}$ $\therefore t = \log_e 3$	2	$\frac{1}{2}$ for $=0$
iii)	$d = \int v \, dt$ $= \int_0^{\log_e 3} 36 - 4e^{2t} \, dt$ ✓ $\frac{1}{2}$ $= \left[36t - \frac{4e^{2t}}{2}\right]_0^{\log_e 3}$ ✓ $\frac{1}{2}$ $\frac{1}{2}$ $= \left(36(\log_e 3) - \frac{4e^{2\log_e 3}}{2}\right) - \left(36(0) - \frac{4e^{2(0)}}{2}\right)$ $= \left(36 \log_e 3 - 2e^{\log_e 9}\right) - (0 - 2)$ $= 36 \log_e 3 - 2 \times 9 + 2$ $= (36 \log_e 3 - 16)$ m $\frac{1}{2}$ ≈ 23.55 m	2	
iv)	$a = \frac{dv}{dt}$ $= -8e^{2t}$ ① ✓ $V = 36 - 4e^{2t}$ $4e^{2t} = 36 - V$ $\frac{1}{2}$ $e^{2t} = \frac{36-V}{4}$ ② sub ② into ① $a = -8\left(\frac{36-V}{4}\right)$ $\frac{1}{2}$ $= -2(36 - V)$	2	

OR 15 c iii)

$V = 36 - 4e^{2t}$

$d = \int 36 - 4e^{2t} \, dt$ $\frac{1}{2}$

$d = 36t - 2e^{2t} + c$

at $t=0$ $d=0$

$0 = 36(0) - 2e^{2(0)} + c$

$c = 2$ ✓ $\frac{1}{2}$

at $t = \log_e 3$ $\frac{1}{2}$

$d = 36(\log_e 3) - 2e^{2\log_e 3} + 2$

$= 36(\log_e 3) - 2 \times 9 + 2$

$= 36 \log_e 3 - 16.$ $\frac{1}{2}$

Qn	16 Solutions	Marks	Comments: Criteria

(a) (i)

$R = 6t + 5$

$D = \int R \, dt$

$= \int 6t + 5 \, dt$

$\therefore D = \frac{6t^2}{2} + 5t + c$ ✓

when $t = 0$ $D = 150$

$150 = 3(0)^2 + 5(0) + c$

$\therefore c = 150$ ✓

so $D = 3t^2 + 5t + 150$ $\frac{1}{2}$

when $t = 8$

$D = 3(8)^2 + 5(8) + 150$ $\frac{1}{2}$

$= 382$ mL

Marks: 3

(ii) when $D = 900$

$900 = 3t^2 + 5t + 150$ ✓

$3t^2 + 5t + 750 = 0$

$(3t + 50)(t - 15) = 0$

$\therefore t = 15$ since $t \geq 0$ ✓

Marks: 2

Comments: $-\frac{1}{2}$ if give $t = -\frac{50}{3}$ as an answer.

(b) (i)

$6\pi = r\theta$

$\frac{6\pi}{r} = \theta$ $\frac{1}{2}$

since $6 < r < 12$

if $r = 6$ $\theta = \frac{6\pi}{6} = \pi$ $\frac{1}{2}$

if $r = 12$ $\theta = \frac{6\pi}{12} = \frac{\pi}{2}$ $\frac{1}{2}$

$\therefore \frac{\pi}{2} < \theta < \pi$ so θ is obtuse $\frac{1}{2}$

Marks: 2

(ii) if $\theta = \frac{3\pi}{4}$

$6\pi = \frac{3\pi}{4} r$

$\therefore r = 8$ ✓

$A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2}(8)^2 \left(\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right)$ $\frac{1}{2}$

$= 32\left(\frac{3\pi}{4} - \frac{1}{\sqrt{2}}\right)$ $\frac{1}{2}$

$= 24\pi - 16\sqrt{2}$

$= 52.77$

Marks: 2

Comments: 1 for $r = 8$; $+\frac{1}{2}$ for substit; $\frac{1}{2}$ for correct simplified answer

Qn	16 cont Solutions	Marks	Comments: Criteria

(c) (i)

$y = \frac{10}{3 + 2\sin x}$ $\qquad 0 \leq x \leq 2\pi$

$y = 10(3 + 2\sin x)^{-1}$

$\frac{dy}{dx} = 10\cos x(3 + 2\sin x)^{-2} \times -2$

$= \frac{-20\cos x}{(3 + 2\sin x)^2}$

Marks: 1

(ii) for stationary pts $\frac{dy}{dx} = 0$

$\frac{-20\cos x}{(3 + 2\sin x)^2} = 0$

$-20\cos x = 0$

$\cos x = 0$ ✓

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{2}$ $\quad y = \frac{10}{3 + 2\sin\frac{\pi}{2}} = \frac{10}{3+2} = 2$

x	$\frac{\pi}{2}^-$	$\frac{\pi}{2}$	$\frac{\pi}{2}^+$
$\frac{dy}{dx}$	$-$	0	$+$
	\	—	/

$\left(\frac{\pi}{2}, 2\right)$ is a minimum $\frac{1}{2}$ $\frac{1}{2}$

$x = \frac{3\pi}{2}$ $\quad y = \frac{10}{3 + 2\sin\frac{3\pi}{2}} = \frac{10}{3 + 2(-1)} = 10$

x	$\frac{3\pi}{2}^-$	$\frac{3\pi}{2}$	$\frac{3\pi}{2}^+$
$\frac{dy}{dx}$	$+$	0	$-$
	/	—	\

$\therefore \left(\frac{3\pi}{2}, 10\right)$ is a maximum $\frac{1}{2}$ $\frac{1}{2}$

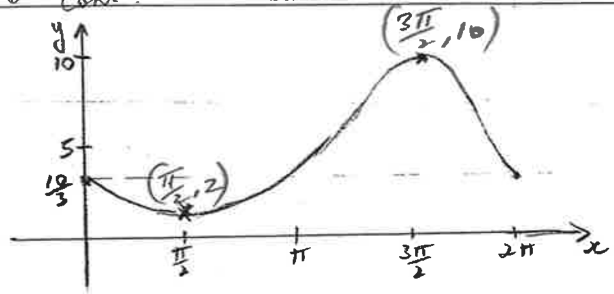
Marks: 3

Comments: if only points $1\frac{1}{2}$; $1\frac{1}{2}$ for nature only; $2\frac{1}{2}/3$ if don't find points; $2\frac{1}{2}$ for wrong min or max

(iii)

at $x = 0$ $\quad y = \frac{10}{3 + 2\sin 0} = \frac{10}{3}$

at $x = 2\pi$ $\quad y = \frac{10}{3 + 2\sin 2\pi} = \frac{10}{3}$

Qn	Solutions	Marks	Comments: Criteria
16 cont.		2	$-\frac{1}{2}$ for wrong or no endpoints. -1 for wrong shape $-\frac{1}{2}$ for wrong or no scale